

Московский физико-технический институт
(Государственный университет)
Департамент молекулярной и биологической физики

Физические методы исследования

Лекция 2

Общие проблемы измерений Электрические цепи систем детектирования

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г. Долгопрудный, 10 февраля 2023 г.

Сложение и «распространение» ошибок

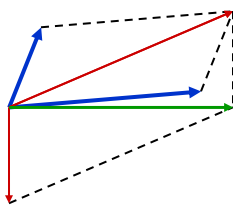
$$\sigma_w^2 = \sigma_x^2 + \sigma_y^2$$

Погрешность среднего

$$W = x_1 + x_2 + \dots + x_n$$

$$\sigma_w^2 = \sigma_1^2 + \sigma_2^2 + \dots + \sigma_N^2 = N \sigma_x^2$$

$$\sigma_{(x)} = \frac{\sigma_x}{\sqrt{N}}$$

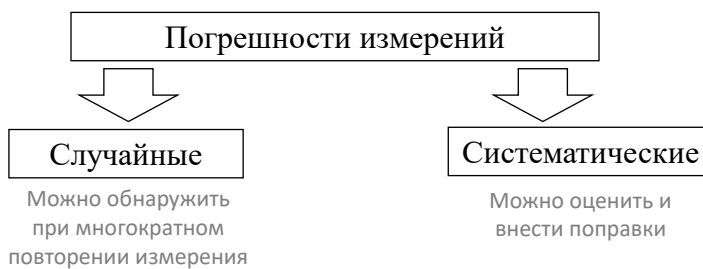


Косвенные измерения

$$x = f(\bar{p}, \bar{q}, \bar{r}, \dots) + \Delta f$$

$$\Delta f = \frac{\partial f}{\partial p} \Delta p + \frac{\partial f}{\partial q} \Delta q + \frac{\partial f}{\partial r} \Delta r + \dots$$

$$\sigma_x^2 = \left(\frac{\partial f}{\partial p}\right)^2 \sigma_p^2 + \left(\frac{\partial f}{\partial q}\right)^2 \sigma_q^2 + \left(\frac{\partial f}{\partial r}\right)^2 \sigma_r^2 + \dots$$



Причины возникновения случайных погрешностей:

- Несовершенство измерительного оборудования;
- Особенности методики измерения, в т.ч. человеческий фактор;
- Особенности объекта измерения, в т.ч. случайный характер исследуемого явления;
- Особенности используемой теоретической модели;
- Шумы;
- Помехи;
- и т.д.

Литература к лекции 1

Максимычев А.В. Физические методы исследования. 1. Погрешности измерений. М., МФТИ, 2006.

Тейлор Дж. Введение в теорию ошибок. М. Мир, 1985.

Гмурман В.Е. Теория вероятностей и математическая статистика. М. Высшая школа, 2002.

Х.-И. Кунце Методы физических измерений. М. Мир, 1989.

Н.С. Маркин Основы теории обработки результатов измерений, М.: Издательство стандартов, 1991. — 176 с.

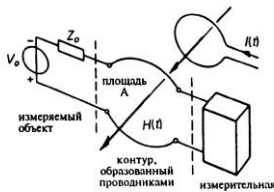
Харт Х. Введение в измерительную технику. М. Мир, 1999.

М.И. Пергамент Методы исследований в экспериментальной физике. Изд-во Интеллект, 2010

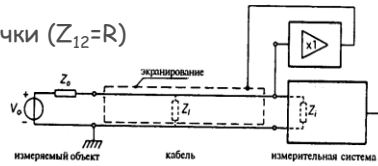
Помехи

- Космические
- Атмосферные
- Промышленные

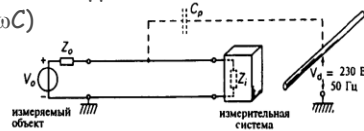
Помехи можно устранить
организационными или
техническими мерами



Токи утечки ($Z_{12}=R$)



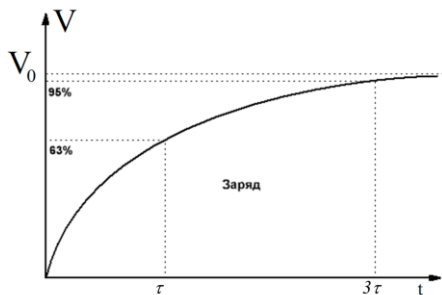
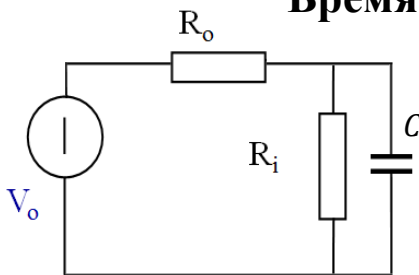
Емкостная наводка
($Z_{12}=1/i\omega C$)



Индуктивная наводка
($Z_{12}=i\omega L$)

$$V_d = -\frac{d\Phi}{dt} = -\frac{d}{dt} \int_A H dA$$

Время измерения



$$I_R = I_0 - I_C$$

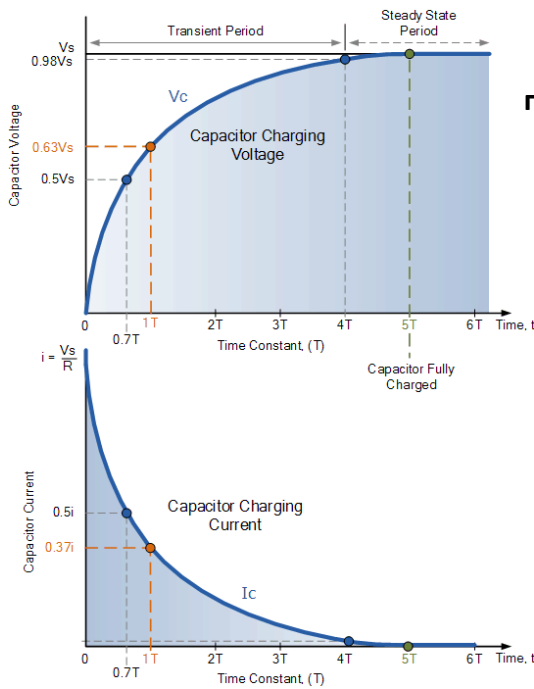
$$q = CV_i; I_C = \frac{dq}{dt} = C \frac{dV_i}{dt}$$

$$V_i = I_R R_i = R_i (I_0 - I_C) = V_{i\infty} - R_i C \frac{dV_i}{dt}$$

$$\frac{dV_i}{V_i - V_{i\infty}} = -\frac{dt}{R_i C}$$

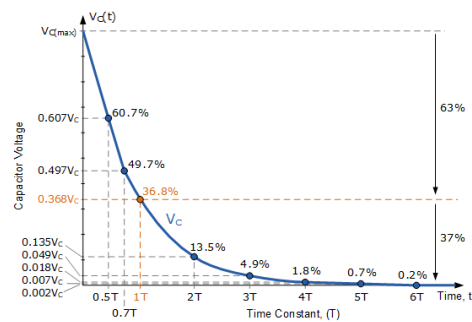
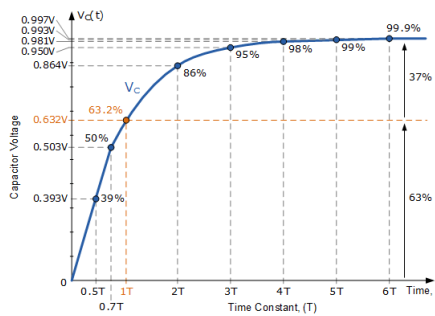
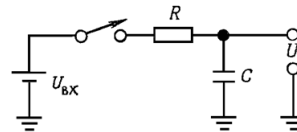
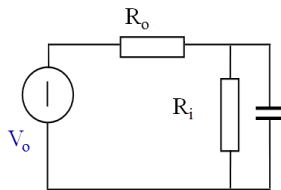
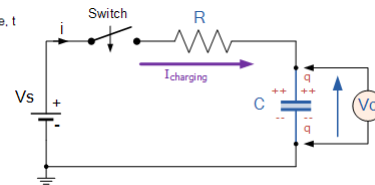
$$V_i = V_{i\infty} \left(1 - \exp \left\{ -\frac{t}{\tau} \right\} \right)$$

$$\tau = R_i C$$



Вольт-амперная характеристика процесса зарядки RC-цепочки

$$V_i = V_{i\infty} \left(1 - \exp \left\{ -\frac{t}{RC} \right\} \right)$$



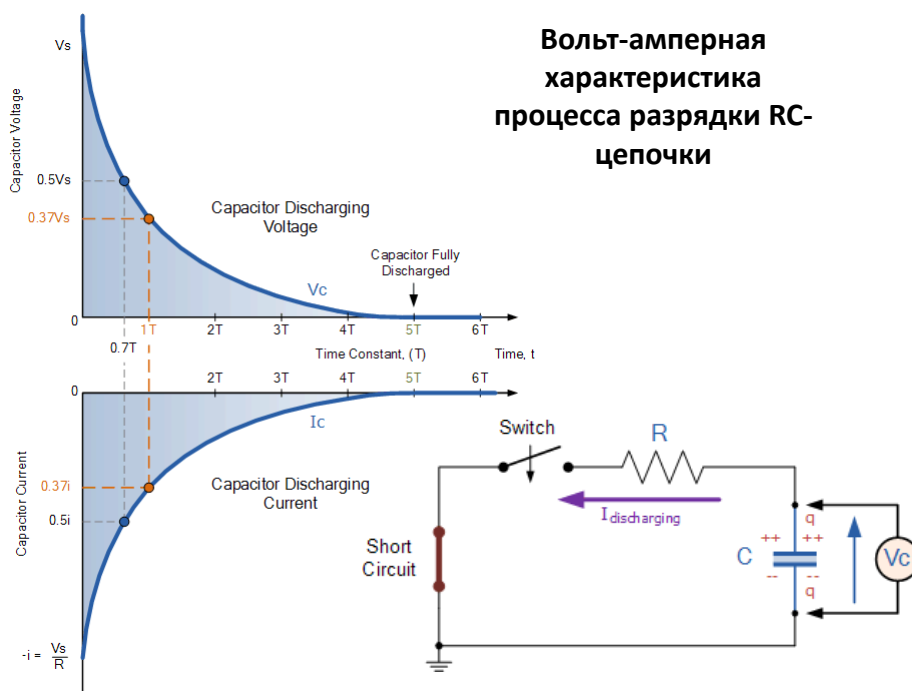
$$V_i = V_{i\infty} \left(1 - \exp \left\{ -\frac{t}{T} \right\} \right)$$

$$T = R_i C$$

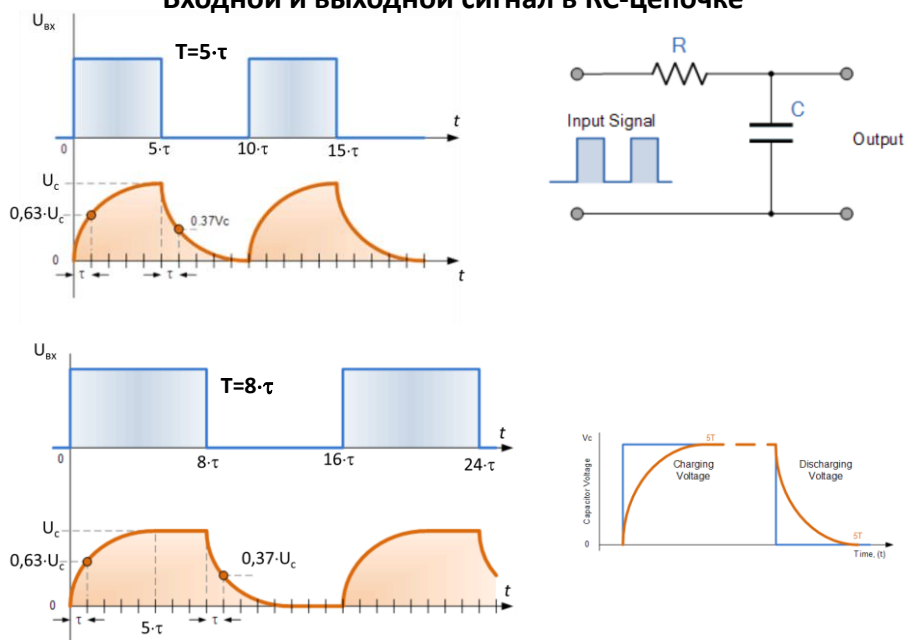
$$I = C \frac{dU}{dt} = -\frac{U_{BX} - U}{R}$$

$$U = U_{BX} + Ae^{-t/RC}$$

Вольт-амперная характеристика процесса разрядки RC-цепочки

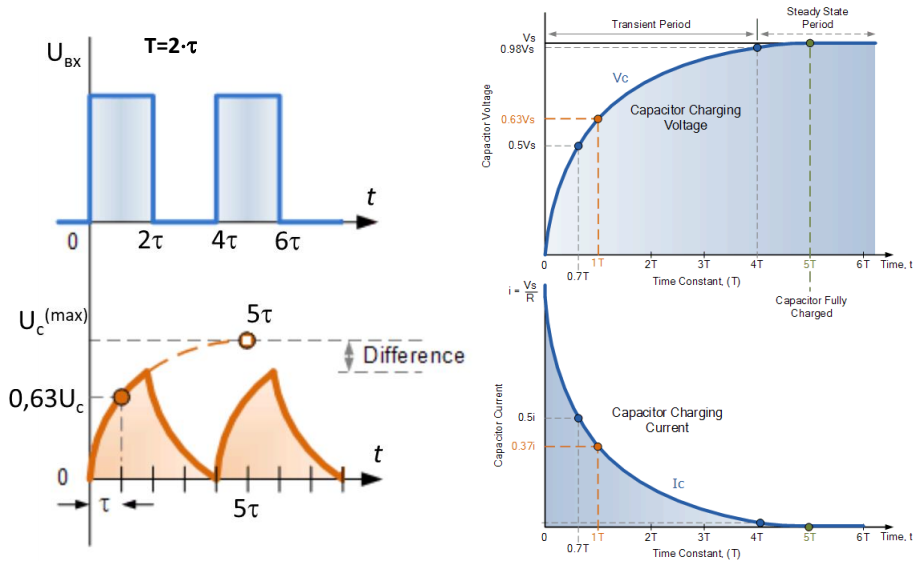


Входной и выходной сигнал в RC-цепочке

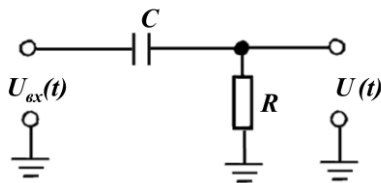


https://www.electronics-tutorials.ws/rc/rc_3.html

Входной и выходной сигнал в RC-цепочке «Укороченный» импульс ($T < 5 \cdot \tau$)



Дифференцирующая цепочка



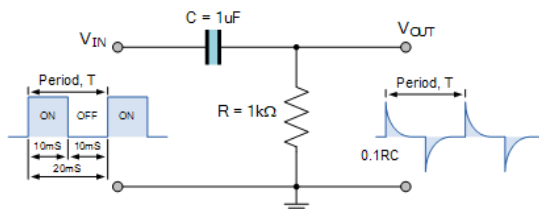
Напряжение на конденсаторе C равно $U_{BX} - U$, поэтому

$$I = C \frac{d(U_{BX} - U(t))}{dt} = \frac{U(t)}{R}$$

R и C выберем так, что R и C – достаточно малые и $\frac{dU}{dt} \ll \frac{dU_{BX}}{dt}$

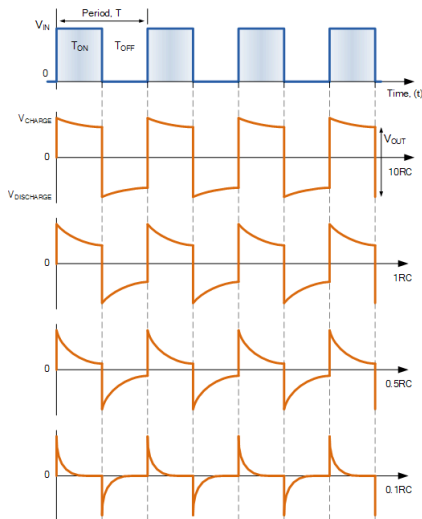
$$C \frac{dU_{BX}}{dt} = \frac{U(t)}{R} \quad U(t) = RC \frac{dU_{BX}(t)}{dt}$$

RC Differentiator Example



So, by having an RC value of one tenth the pulse width (and in our example above this is $0.1 \times 10 \text{ ms} = 1 \text{ ms}$) or lower we can produce the required spikes at the output, and the lower the RC time constant for a given pulse width, the sharper the spikes. Thus, the exact shape of the output waveform depends on the value of the RC time constant.

RC Differentiator Output Waveforms



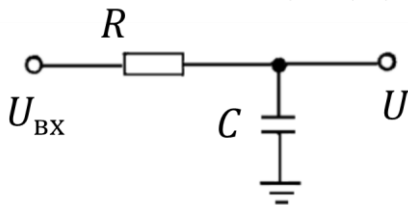
Then we can see that the shape of the output waveform depends on the ratio of the pulse width to the RC time constant. When RC is much larger (greater than $10RC$) than the pulse width the output waveform resembles the square wave of the input signal. When RC is much smaller (less than $0.1RC$) than the pulse width, the output waveform takes the form of very sharp and narrow spikes as shown above.

So, by varying the time constant of the circuit from $10RC$ to $0.1RC$ we can produce a range of different wave shapes. Generally, a smaller time constant is always used in RC differentiator circuits to provide good sharp pulses at the output across R. Thus, the differential of a square wave pulse (high dv/dt step input) is an infinitesimally short spike resulting in an RC differentiator circuit.

Let's assume a square wave waveform has a period, T of 20mS giving a pulse width of 10mS (20mS divided by 2). For the spike to discharge down to 37% of its initial value, the pulse width must equal the RC time constant, that is $RC = 10mS$. If we choose a value for the capacitor, C of 1uF, then R equals 10kΩ.

For the output to resemble the input, we need RC to be ten times ($10RC$) the value of the pulse width, so for a capacitor value of say, 1uF, this would give a resistor value of: 100kΩ. Likewise, for the output to resemble a sharp pulse, we need RC to be one tenth ($0.1RC$) of the pulse width, so for the same capacitor value of 1uF, this would give a resistor value of: 1kΩ, and so on.

Интегрирующая цепочка



Напряжение на резисторе R равно $U_{BX} - U$

Тогда

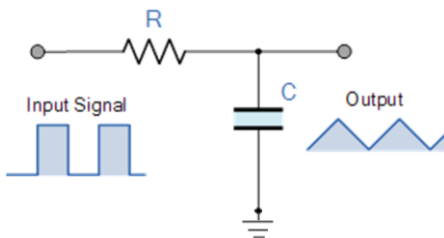
$$I = C \frac{dU}{dt} = \frac{U_{BX} - U}{R}$$

Если выполняется условие $U \ll U_{BX}$ за счет большого значения произведения RC , то получим

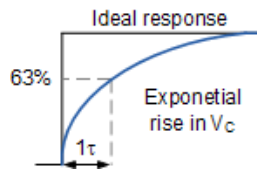
$$C \frac{dU}{dt} \approx \frac{U_{BX}}{R}$$

Что даёт

$$U(t) = \frac{1}{RC} \int_0^t U_{BX}(\tau) d\tau + const$$

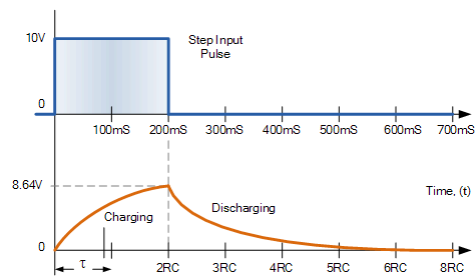
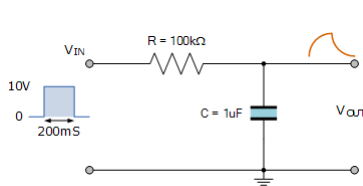


Single Pulse RC Integrator

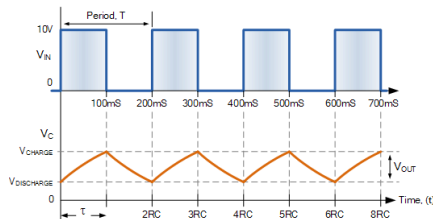
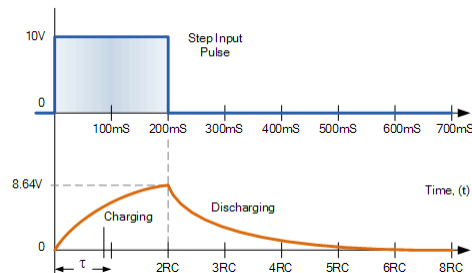
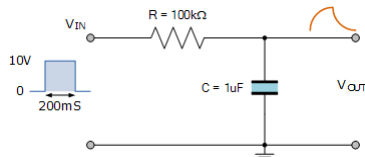


Capacitor Charging $V = V_{\infty} \left(1 - \exp\left\{-\frac{t}{\tau}\right\}\right)$
 $\tau = R_i C$

Capacitor Discharging $U = U_{BX} + U \exp\left\{-\frac{t}{RC}\right\}$



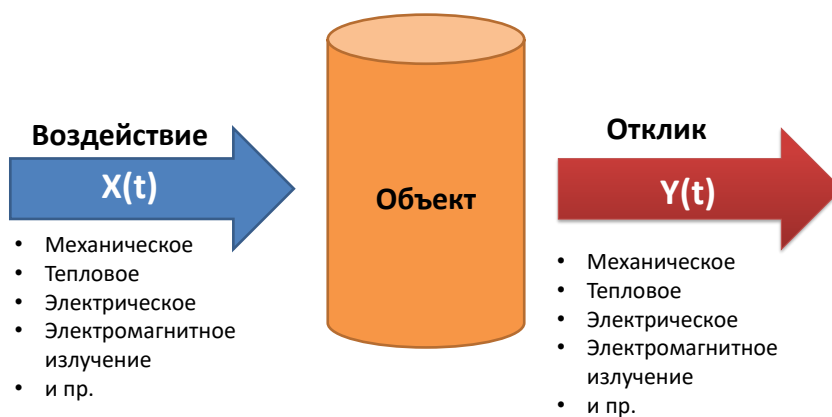
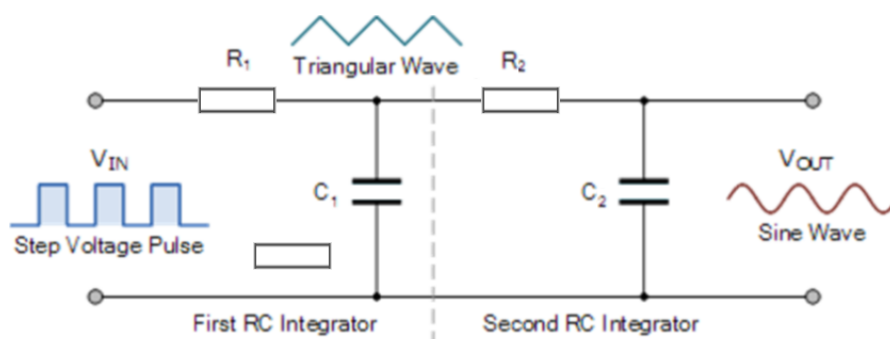
RC Integrator Circuit Example



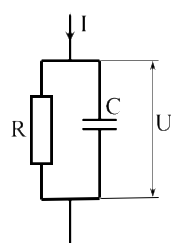
So for a continuous pulse input, the correct relationship between the periodic time of the input and the RC time constant of the circuit, integration of the input will take place producing a sort of ramp up, and then a ramp down output. But for the circuit to function correctly as an integrator, the value of the RC time constant has to be large compared to the input's periodic time. That is $RC \gg T$, usually 10 times greater.

This means that the magnitude of the output voltage (which was proportional to $1/RC$) will be very small between its high and low voltages severely attenuating the output voltage. This is because the capacitor has much less time to charge and discharge between pulses but the average output DC voltage will increase towards one half magnitude of the input and in our pulse example above, this will be 5 volts ($10/2$).

RC Integrator as a Sine Wave Generator



Амплитудно-частотная характеристика

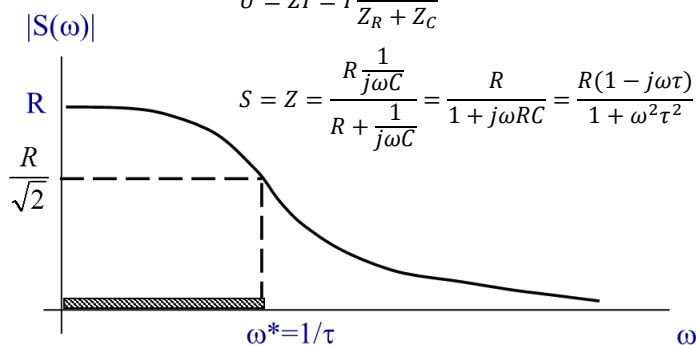


Входной сигнал: $X \equiv I$;
Выходной сигнал: $Y \equiv U$;

Функция преобразования: $S = \frac{Y}{X}$

$$U = ZI = I \frac{Z_R Z_C}{Z_R + Z_C}$$

$$S = Z = \frac{R \frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{R}{1 + j\omega RC} = \frac{R(1 - j\omega\tau)}{1 + \omega^2\tau^2}$$



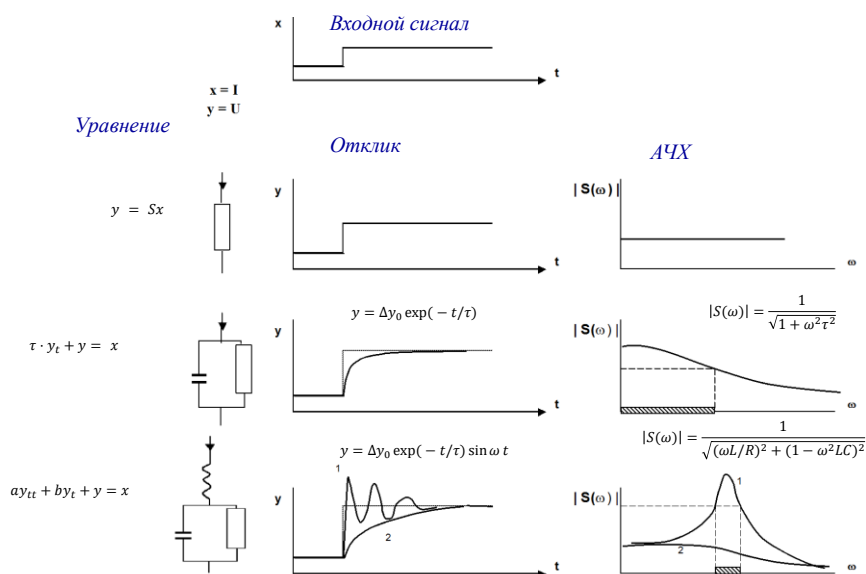
АЧХ $\equiv |S(\omega)|$

$$|S(\omega)| = \frac{R}{\sqrt{1 + \omega^2\tau^2}}$$

$$Re(S) = \frac{R}{1 + \omega^2\tau^2}$$

$$Im(S) = -\frac{R\omega\tau}{1 + \omega^2\tau^2}$$

АЧХ и функция отклика

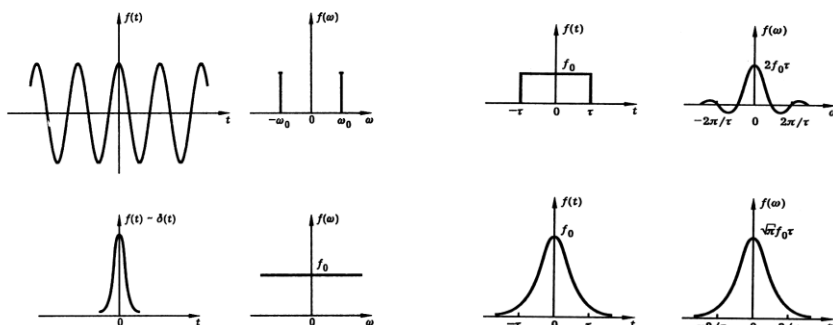


Преобразование Фурье

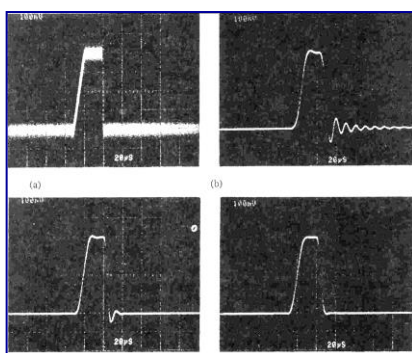
$$F(\omega) = \int_{-\infty}^{+\infty} f(t) e^{-i\omega t} dt$$

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(\omega) e^{i\omega t} d\omega$$

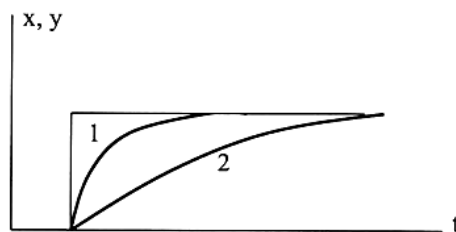
$$\Delta\omega \cdot \Delta t \sim 1$$



Полоса пропускания и время отклика



1 - широкополосная система



2 - узкополосная система

$$\Delta\omega \cdot \Delta t \sim 1$$

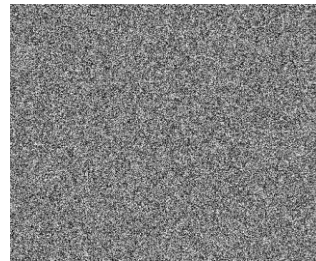
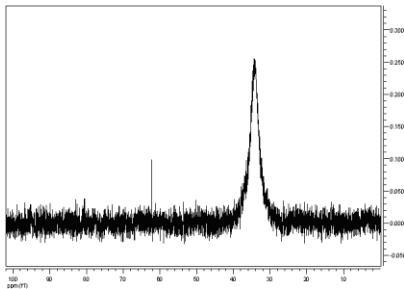
Шумы

$$С П М Ш \quad N \equiv \frac{dP_N}{df}$$

$$P_N = \int_{\Delta\omega} N(\omega) d\omega$$



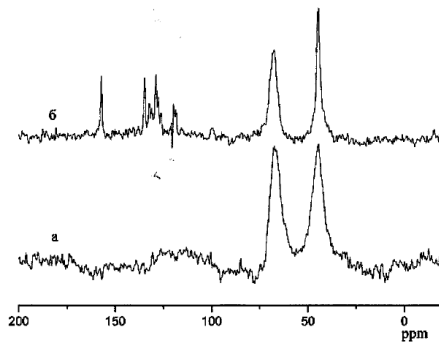
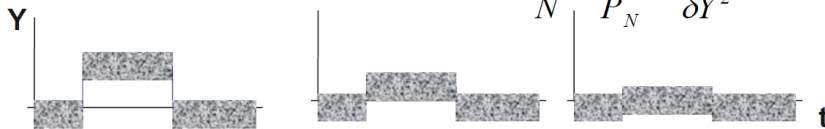
$$S/N = \frac{P_S(\Delta\omega)}{P_N(\Delta\omega)}$$



Характеристики измерений

Отношение сигнал/шум

$$\frac{S}{N} = \frac{P_S}{P_N} = \frac{Y^2}{\delta Y^2}$$

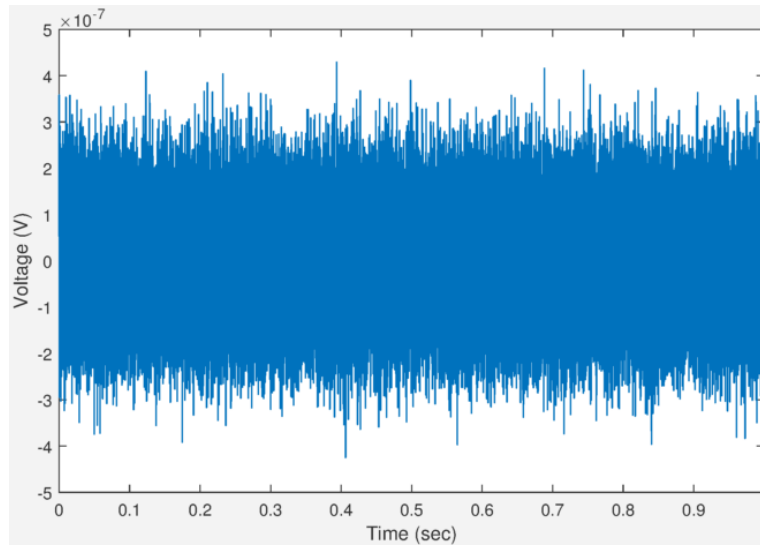


$$I(\text{Бел}) = \lg\left(\frac{P_2}{P_1}\right)$$

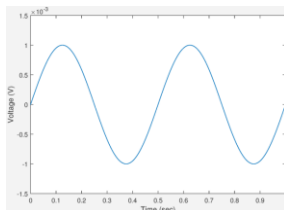
$$I(\text{дБ}) = 10 \cdot \lg\left(\frac{P_2}{P_1}\right) =$$

$$= 10 \cdot \lg\left(\frac{V_2^2}{V_1^2}\right) = 20 \lg\left(\frac{V_2}{V_1}\right)$$

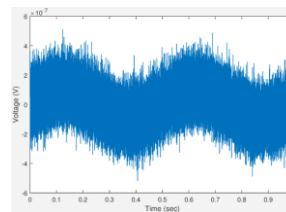
Electrical Noise



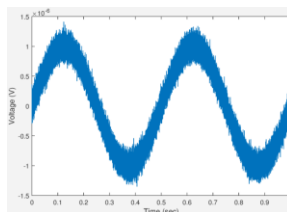
Electrical Noise



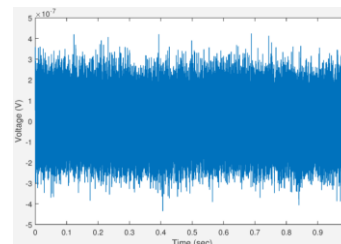
Electrical Noise Not a Problem



Signal Just Bumps Noise Up and Down

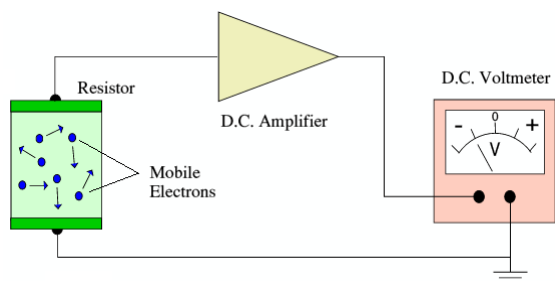


Electrical Noise Very Noticeable

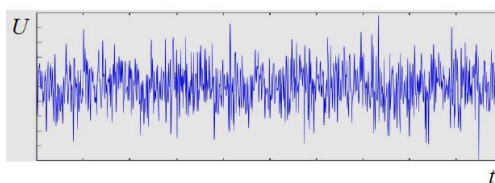


Signal Lost in the Noise

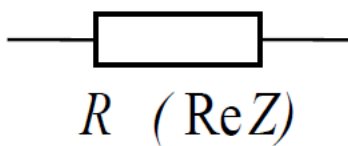
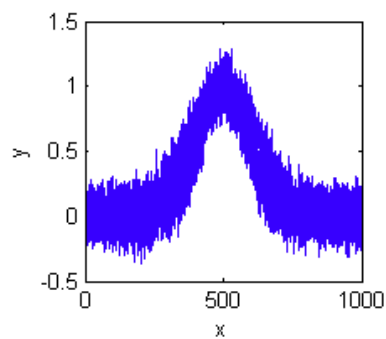
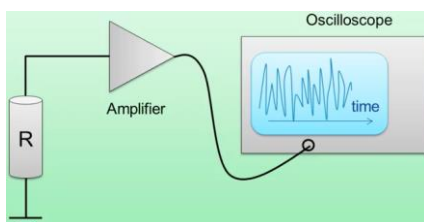
Тепловой шум (шум Найквиста)



Jonson Noise or Nyquist Noise



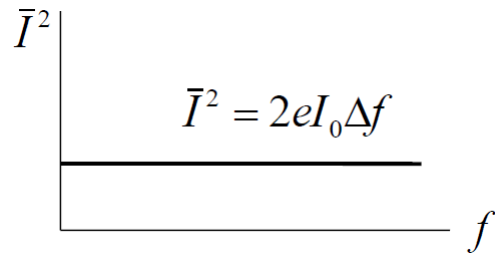
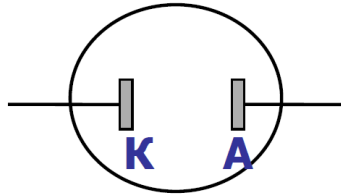
Спектрально независимый «белый» шум



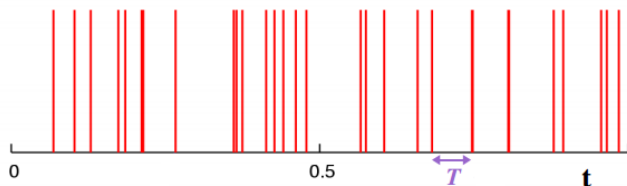
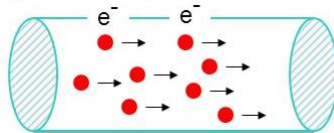
$$\overline{U^2} = 4kTR\Delta f$$

f

Неравновесный дробовой шум



Shot noise in electron transport



Proposed by Schottky to measure the electron charge in 1918

Литература к лекции 2

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